

**FUTURE MISSIONS STUDIES**  
**COMBINING SCHATTEN'S SOLAR ACTIVITY PREDICTION MODEL**  
**WITH A CHAOTIC PREDICTION MODEL**

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## SECTION 1 - INTRODUCTION: CHAOS VERSUS STOCHASTICITY

Although atmospheric dynamics are governed by the same laws of physics as planetary motion, we still forecast weather in terms of probabilities. Because no clear relationship exists between cause and effect in atmospheric physics, atmospheric phenomena seem random, or stochastic. Yet, until recently, we had little reason to doubt that, at least in principle, weather is ultimately predictable. It was assumed that we need only gather and process enough information. Recently, a striking discovery changed our perspective: Simple deterministic systems with only a few degrees of freedom can generate random behavior.

When apparent random behavior is fundamental to the nature of a system, such that no amount of information gathering will make the system predictable, that system is said to be chaotic. Perhaps paradoxically, chaos is generated by fixed rules that do not themselves involve any elements of chance. In principle, the future condition of a dynamic system is completely determined by present and past conditions. In practice, however, amplification of small initial uncertainties makes a system with short-term predictability unpredictable in the long term.

In the list that follows, we highlight some of the major points in the emerging science of chaos.

- Chaos is orderly: Randomness has an underlying geometric form.
- The discovery of chaos has created a new paradigm in scientific modeling. On one hand, it implies new fundamental limits on prediction. On the other hand, the determinism inherent in chaos implies that many random and complex phenomena are more predictable than previously thought.

- Chaotic theory lends order to such diverse systems as Earth weather, the Sun, the human brain and heart, and even the economy.

- The existence of random behavior in very simple systems has motivated a reexamination of the sources of randomness even in highly complex systems such as weather.

- Chaotic theory opposes the determinism of the 18th Century philosopher-mathematician Pierre Simon de Laplace, whose ideas greatly influenced the direction of modern science. Laplace believed that, given the position and velocity of every particle in the universe, one could predict the future for the rest of time. The philosophy of determinism holds that human behavior is predictable, given enough data, and that free will is only an illusion.

- Twentieth Century science has seen the downfall of Laplacian determinism in response both to the Heisenberg uncertainty principle of quantum mechanics and to the idea of sensitive dependence on initial conditions (discussed in Section 2). These ideas have helped us to understand why some apparently simple systems behave unpredictably, even though they are subject to the same laws of motion that allow us to predict the motion of planets, for example, precisely. A balloon filled with air and then released is such a system.

- The Soviet physicist Lev D. Landau was the first who attempted to study turbulence in the 1930s. He maintained that motion of a turbulent fluid includes many independent oscillations. However, later research has contradicted this idea, demonstrating random behavior even in very simple systems, such as a coin toss.

- Early in the 20th Century, Henri Poincare questioned the notion of determinism. Because of a growing interest in quantum mechanics, however, the work of

classical physicists such as Poincare was largely ignored. Only recently have his ideas, which form the foundation of modern chaotic theory, been seriously explored by other scientists.



## SECTION 2 - SOLAR ACTIVITY PREDICTION

Interest in solar activity has grown in the past two decades for many reasons. First, new evidence suggests a correlation between solar activity and weather on Earth (van Loon and Labitzke, 1988), although such a correlation has not yet been convincingly established (Kerr, 1990). In fact, we have evidence of the coincident occurrences of the Maunder Minimum, a period of little or no solar activity occurring from 1645 to 1715, and the "Little Ice Age," a period of abnormally cold weather (Bray, 1971). Second, solar activity is also studied by astronomers concerned only with the Sun itself (Brandt, 1970). Third, and of greatest importance to flight dynamics, solar activity changes the atmospheric density, which has important implications for spacecraft trajectory and lifetime prediction (Walterscheid, 1989).

Because of the seemingly random nature of solar activity, it has generally been assumed that the underlying physics must necessarily be complex as well. As a result, researchers have turned toward statistical models to predict solar dynamics (Withbroe, 1989, and references therein). However, new developments in chaos and nonlinear dynamics have demonstrated that random behavior is not always due to complexity but rather to sensitive dependence on initial conditions, which can sometimes cause even simple systems to become chaotic. This view would allow us to model the behavior of a chaotic system in terms of some invariants directly extractable from system dynamics, without reference to any underlying physics. Using chaos theory, we would be able to predict short-term activity more accurately than with statistical methods, but chaos theory puts a fundamental limit on long-term predictions. The philosophy behind our approach is introduced in the next section.



### SECTION 3 - LOW DIMENSIONAL CHAOS VERSUS COMPLEXITY

In our previous communications (Ashrafi, January 1991a, January 1991b; Ashrafi and Roszman, May 1991, June 1991a, June 1991b, July 1991), we showed how thinking in terms of deterministic dynamics and assuming that randomness arises out of chaos rather than complexity lead to new approaches to forecasting and nonlinear modeling.

Until recently, it was usually assumed that randomness was caused by extreme complication, that is, the presence of many irreducible degrees of freedom. This naturally led to Kolmogorov's theory of random processes, which he defined in terms of the joint probability distribution. The process is deterministic if there is some value  $d$  (distribution order) for which the probability density approaches a delta function.

Many people speak of random processes as though they were a fundamental source of randomness. This idea is misleading. The theory of random processes is an empirical technique for coping with inadequate information; it makes no statements about the causes of randomness. As far as we know, the only truly fundamental source of randomness is the uncertainty principle of quantum mechanics; everything else is deterministic, at least in principle. Nonetheless, we call many phenomena, such as fluid turbulence, random, even though they have no obvious connection to quantum mechanics. It has traditionally been assumed that the apparent randomness of these phenomena derives solely from their complication.

We will take the practical viewpoint that randomness occurs to the extent that a system's behavior is unpredictable, which usually depends on the available information. With more data or more accurate observations, a phenomenon that

had previously seemed random might become more predictable and, hence, less random. Therefore, we believe that randomness is subjective. Furthermore, randomness is a matter of degree; that is, some systems are more predictable than others (e.g., solar activity is more predictable than geomagnetic activity).

As originally pointed out by Poincare, many of the classic examples of randomness are not complicated. The dynamics of a flipping coin, for example, involve only a few degrees of freedom. This randomness comes from sensitive dependence on initial conditions--a small perturbation causes a much larger effect at a later time, making prediction difficult. When sensitive dependence on initial conditions occurs in a sustained way, we call the result chaos. Since chaos is defined in the context of deterministic dynamics, in some very strict sense it is incorrect to say that chaos is random; ultimately, uncertainty originates from something external to the dynamics, such as measurement error or external noise. However, sensitive dependence exaggerates uncertainty, so that small uncertainties turn into large ones. Because chaos amplifies noise exponentially, any uncertainty at all is amplified to macroscopic proportions in finite time, and short-term determinism becomes long-term randomness: Chaos creates randomness by strongly amplifying what we don't know.

Chaotic systems pass many classic "tests" of randomness; for example, some simple chaotic maps produce uncorrelated time series, with  $\langle x_t x_{t+j} \rangle = 0$  unless  $j = 0$ . Furthermore, chaotic trajectories look random. Dissipative dynamic systems often have the property that undisturbed trajectories approach a subset of the state space, called an attractor. Fluid flows, for example, have an effective infinite dimensional state space but can have low dimensional attractor.

Thus, we should not distinguish chaos from randomness but should instead distinguish systems with low dimensional attractor from those with high dimensional attractor. With many degrees of freedom, the statistical approach is probably as good as any. However, if random behavior comes from low dimensional chaos, we can make much more accurate forecasts than those made using statistical models. Furthermore, the resulting chaotic models can give useful diagnostic information about the nature of the underlying dynamics, aiding the search for a description in terms of first principles.

One of the leading theories proposed over the last several decades assumes that the Sun behaves as a hydromagnetic dynamo (Gilman, 1985). Many dynamo-based models of varying degrees of complexity have been proposed (Gilman, 1986). Zeldovich and Ruzmaikin (1987) have developed a low dimensional solar dynamo model, a concept first discussed by Ruzmaikin (1981).

Through some simple canonical transformations, we have been able to transform dynamo equations into established Lorenz (1963) equations. Lorenz equations are a classic example of equations that exhibit complex chaotic behavior, including intermittency, for a wide range of parameter values. Additionally, Jones et al. (1985) and Weiss (1985, 1988) have considered a model consisting of six differential equations, as opposed to three Lorenz equations. This model also exhibits chaotic behavior and indicates a period-doubling route to chaos. We had independently observed period doubling through a careful study of solar flux power spectral density, an indicator of solar activity (Ashrafi and Roszman, May 1991).

We have determined the correlation dimension of solar flux time series to be about 2.5, which indicates that only three

independent variables are needed to describe its evolution. The Lorenz and Rossler attractors have very similar correlation dimension, also requiring three independent variables to completely describe the system. Using a turbulent version of the dynamo model would allow us to model the long-time evolution of solar cycles with a low dimensional set of ordinary differential equations.

We believe that a turbulent, or chaotic, solar dynamo model may explain the Maunder minimas. Eddy (1976) has argued convincingly against the idea that solar activity was occurring but simply not observed from 1645 to 1715. He concludes that, very likely, solar activity was actually lacking during that period. A period of little or no solar activity could be explained as resulting from the phenomenon of intermittency (Ashrafi, August 1990, personal communication), which occurs when a system alternates between periods of laminar and chaotic behavior.

Pomeau and Manneville (1979) have established that intermittency does indeed occur in the Lorenz system. A detailed analysis of intermittency is given by Schuster (1989). Therefore, the Maunder Minimum may have evolved on the same attractor but in a region where the solar activity as a function of time is very regular. If this view is correct, we would expect the solar cycle to alternate between chaotic and laminar behavior at irregular intervals. Interestingly enough, we have evidence of other minima occurring at certain times throughout history, for example, the Sporer minimum, which occurred between 1460 and 1550 (Eddy, 1976). The idea of intermittency brings a consistency to Schatten's solar dynamic model (Schatten, 1978, 1987) and our chaotic model (Ashrafi, July 1991; unpublished manuscripts a, b, and c). We believe that no complicated mechanism need be invoked to describe the qualitatively diverse behavior observed in the solar cycle

over the past 300 or, for that matter, 1000 years. The various behaviors observed are, in our view, simply natural consequences of a chaotic system.

Feynman and Gabriel (1990) have recently analyzed 1500 years of auroral, geomagnetic, and solar activity and have proved that solar activity does not follow quasi-periodicity, supporting our assertion that solar activity is indeed chaotic.

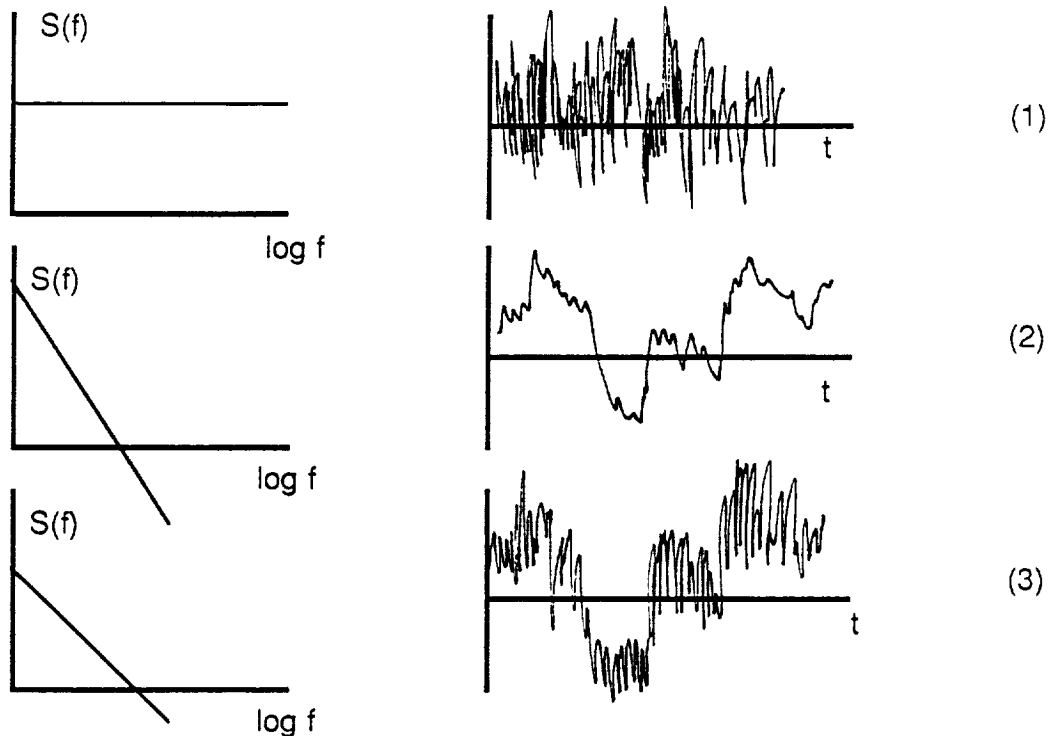


#### SECTION 4 - FRACTAL STRUCTURE IN SOLAR FLUX SIGNAL

Fractal geometry provides both a description and a mathematical model of seemingly complex forms in nature. Shapes and signals found in nature are not easily described by traditional methods. Nevertheless, they often possess a remarkable simplifying invariance under changes of magnification. This statistical self-similarity is the essential quality of fractals in nature: Nature is quantified by a fractal dimension. Fractal shapes are said to be self-similar and independent of scale or scaling. With the use of fractals, iteration of a very simple rule can produce seemingly complex shapes with some highly unusual properties. Unlike Euclidian shapes, these curves have details on all length scales. Fractals remain by far the best approximation of the real world. The fractal dimension determines the relative detail or irregularity at different scales (time or space). The addition of irregularities on smaller and smaller scales raises the dimension. Changes in time, however, have many of the same similarities at different scales and changes in space.

The spectral density,  $S(f)$ , gives an estimate of the mean square fluctuations at frequency  $f$  and, consequently, of the variations over a time scale of order  $1/f$ . Solar flux does have fractal structure, and a direct relationship exists between fractal dimension and logarithmic slope of the spectral density (Figure 3-1).

The following signals are classified with respect to randomness: (1)  $1/f^0$  white noise, the most random signal; (2)  $1/f^2$  Brownian noise, the most correlated of all signals; and (3)  $1/f^\beta$  fractional Brownian (FB) noise ( $0.5 < \beta < 1.5$ ), intermediate between white noise and Brownian noise. Although its origin is, as yet, a mystery, FB noise is the most common signal in nature.



**Figure 4-1. Types of Noise With Their Power Spectra:**  
**(1) White Noise, (2) Brownian Noise, and**  
**(3) Fractional Brownian Noise**

Both  $1/f^0$  white noise and  $1/f^2$  Brownian noise are well understood mathematically. However, to date, no simple mathematical models produce  $1/f^\beta$  FB noise. The spectral density of solar flux has a fractal structure of dimension 2.5. This fractal structure allows us to rescale the time and extend our prediction horizon. One might conclude that now our extended prediction will not have as detailed a structure as our unextended predictions. This is not completely true, for once we have enough data to construct the attractor in the embedding space, the extended predictions are approximately as good as the unextended ones. However, there would come a point  $T_{\text{ultimate}}$  at which, as we rescale the time, we no longer have enough data points to construct the attractor.

## SECTION 5 - COMBINING SCHATTEN'S MODEL WITH OUR CHAOTIC MODEL

K. Schatten (1991) has recently developed a method for combining his prediction model with our chaotic model. The philosophy behind this combined model and his method of combination is explained below.

Schatten's Model (KS). Because KS uses a dynamo to mimic solar dynamics, accurate prediction is limited to long-term solar behavior (10 to 20 years).

Chaotic Model (SA). SA uses the recently developed techniques of nonlinear dynamics to predict solar activity. It can be used to predict activity only up to a horizon. In theory, the chaotic prediction should be several orders of magnitude better than statistical predictions up to that horizon; beyond the horizon, chaotic predictions would theoretically be just as good as statistical predictions. Therefore, chaos theory puts a fundamental limit on predictability.

After embedding the solar flux time series in a state space using the Taken-Packard delay coordinate technique, one can "learn" the induced nonlinear mapping using a local approximation. This will allow us to make short-term forecasting of the future behavior of our time series using information based only on past values. The error estimate of such a technique has already been developed by Farmer and Sidorowich (1987).

$$E \sim C e^{(m+1)KT} N^{-(m+1)/D}$$

where  $E$  = normalized error of prediction ( $0 \leq E \leq 1$ , where zero is perfect prediction, and one is a prediction no better than average)

$m$  = order of local approximation

$K$  = Kolmogorov entropy

$T$  = forecasting window

$N$  = number of data points

$D$  = Dimension of the attractor

$C$  = normalization constant

Using the Farmer-Sidorowich relation, we can find the prediction horizon  $T$  for the zeroth order of local approximation. Any prediction above  $T_{\max}$  is no better than average constant prediction.

$$E(T_{\max}) = 1$$

thus,

for  $m = 0$ ,  $K$  is the largest Lyapunov exponent  $\lambda$ .

Therefore,

$$e^{KT_{\max}} N^{-1/D} \sim 1 \quad \text{or} \quad T_{\max} \sim \frac{\ln(N)}{KD}$$

and

$$T_{\max} \sim \frac{\ln(N)}{\lambda D}$$

For a finite length of data, one has to calculate the local Lyapunov exponent. For  $N = 4090$  point from daily solar flux data and  $\lambda \sim 0.01$  and  $D \sim 2.5$  (like a Lorenz system),  $T_{\max} \sim 70$  days, or about 2 months. For 250 years of averaged monthly data,  $T_{\max} \sim 4$  years. Any prediction beyond the indicated horizons is no better than average value. The

connection between the local and the global Lyapunov exponents has recently been found by Abrabanel and Kennel (March 1991) in a form of power law as

$$\lambda(l) = \lambda_G + \frac{c}{l^v}$$

$$N = \omega l$$

where  $\lambda(l)$  = local Lyapunov exponent

$l$  = length of observed data (observation window)

$v$  = a constant dependent to the dynamic system  
( $0.5 \leq v \leq 1.0$ )

$c$  = a constant dependent to initial conditions of the system

$\lambda_G$  = well-known global Lyapunov exponent

$\omega$  = frequency of data points

Because any data are of finite length, using the Abrabanel-Kennel power law and the Farmer-Sidorowich relation, we can find  $T_{\max}$  as

$$T_{\max} \sim \frac{\ln(l\omega)}{\lambda_G + \frac{c}{l^v}} D$$

This means that as  $l$  increases linearly,  $T_{\max}$  increases logarithmically to a certain asymptotic  $T$  because of the denominator  $C/l^v$  (Figure 5-1).

Therefore, our relation shows that as the asymptote  $T_{\max}$  approaches  $T_0$ , the  $dT_{\max}/dl$  approaches 0, and, thus, we can find what observation window is required for forecasting up to  $T_{\max}$  within some confidence level.

$$\frac{dT_{\max}}{dN} \rightarrow 0 \quad \text{thus } N_0 \sim e^{\frac{x_0(\delta)}{v}} \quad \chi_0(\delta) > 2$$



Combined Models: K. Schatten (1991) has introduced the following method of combining the KS and SA models:

$$f_{\text{pred}} = \text{avg} \times f_{\text{SA}}^{A(t)} \times f_{\text{KS}}^{K(t)}$$

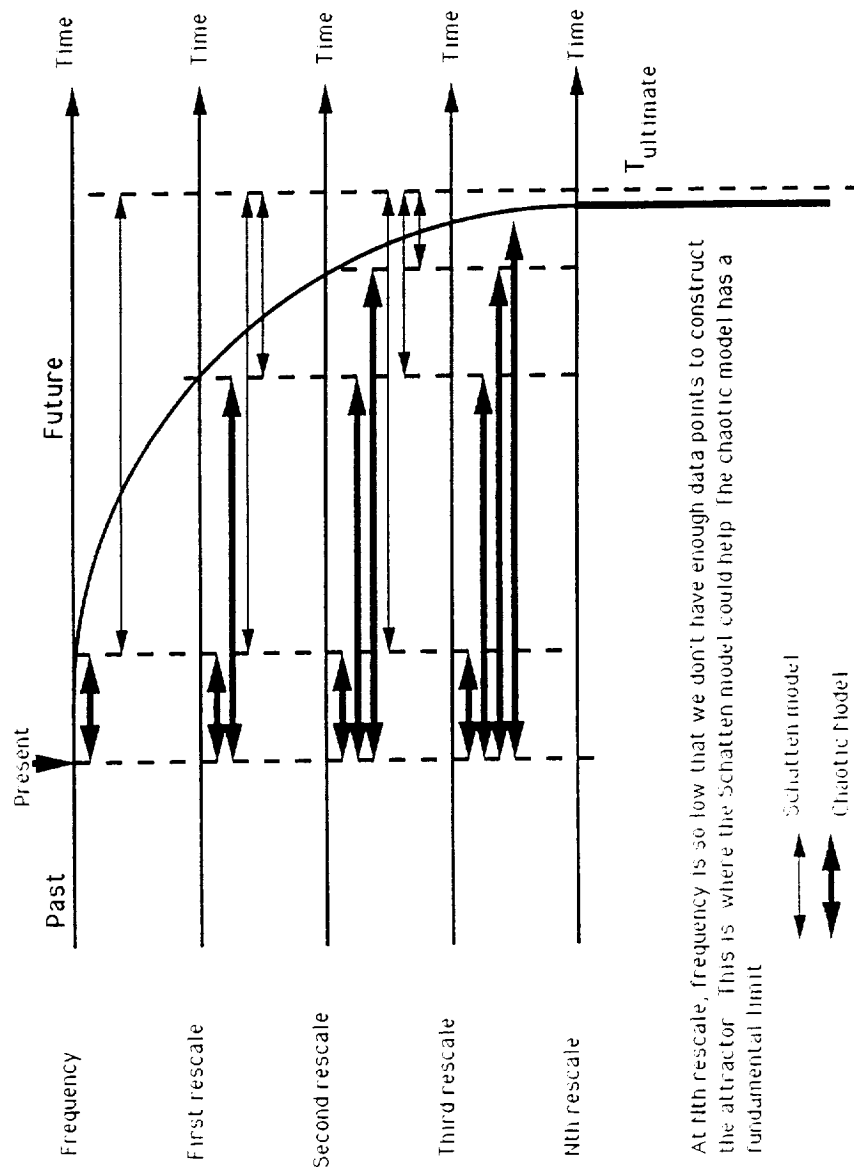
where  $f_{\text{pred}}$  is the prediction of solar flux  
 $\text{KS}$  is the prediction by Schatten's model  
 $\text{SA}$  is the prediction by the chaotic model  
 $\text{avg}$  is a constant  
 $A(t)$  and  $K(t)$  are both time-dependent functions

$$f_{\text{KS}} = \frac{\text{KS}}{\text{avg}} \quad K(t) = 1 - e^{\frac{-t}{\tau}}$$

$$f_{\text{SA}} = \frac{\text{SA}}{\text{avg}} \quad A(t) = e^{\frac{-t}{\tau}}$$

An alternate method of combining the two models is as follows:

1. Calculate the prediction horizon for different frequencies of data. This allows us to extend our horizon at the expense of losing some of the data points.
2. Predict the time series up to the specified horizons.
3. Combine all the predictions, including the Schatten prediction, by minimizing combined variances or by using an appropriate Kernel density, as shown in Figure 5-2.



**Figure 5-2. Schatten Model Versus Chaotic Model**

### Dynamo Equations (Solar Model):

$$I \frac{d\omega}{dt} = T - Mi_b i_c - v\omega$$

$$L_b \frac{di_b}{dt} = (M\omega + B_s) - (B_b + B_s)i_b$$

$$L_c \frac{di_c}{dt} = B_s i_b - (B_c + B_s)i_c$$

where

<b>I</b> :	Moment of inertia	<b><math>\omega</math></b> :	Angular velocity
<b>M</b> :	Mutual inductance	<b><math>L_b</math></b> :	Brush self-inductance
<b><math>L_c</math></b> :	Coil self-inductance	<b><math>B_s</math></b> :	Resistance of shunt
<b><math>B_b</math></b> :	Resistance of brush	<b><math>B_c</math></b> :	Resistance of coil

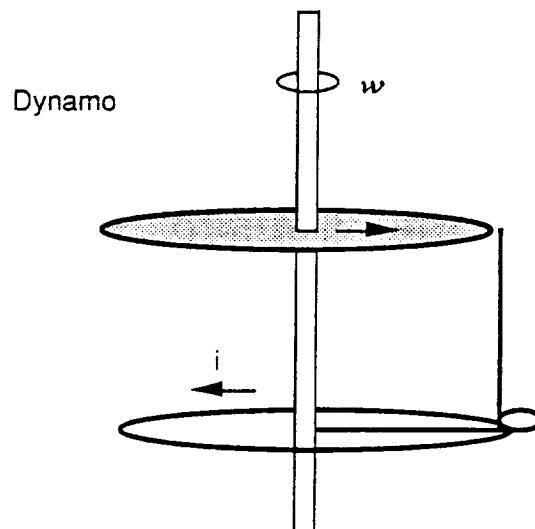


Figure 5-3. Dynamo Equations (Solar Model)

### Our Canonical Transformation of Dynamo Equations:

$$t \rightarrow \tau t$$

$$i_b \rightarrow \alpha y$$

$$i_c \rightarrow \beta x$$

$$\omega \rightarrow -\gamma z + \frac{T}{v}$$

### Resulting Equations of Lorenz:

$$\dot{x} = \sigma y - \sigma x$$

$$\dot{y} = -xy + rx - y$$

$$\dot{z} = xy - bz$$

where

$$\sigma = \left(\frac{B_c + b_s}{L_c}\right) \left(\frac{L_b}{B_b + B_s}\right)$$

$$r = \left(\frac{TM}{L_b v} + \frac{B_s}{L_b}\right) \left(\frac{B_s}{B_c + B_s}\right) \left(\frac{L_b}{B_b + B_s}\right)$$

$$b = \frac{v}{T} \left(\frac{L_b}{B_b + B_s}\right)$$

**Figure 5-4. Canonical Transformation and Lorenz Equations**

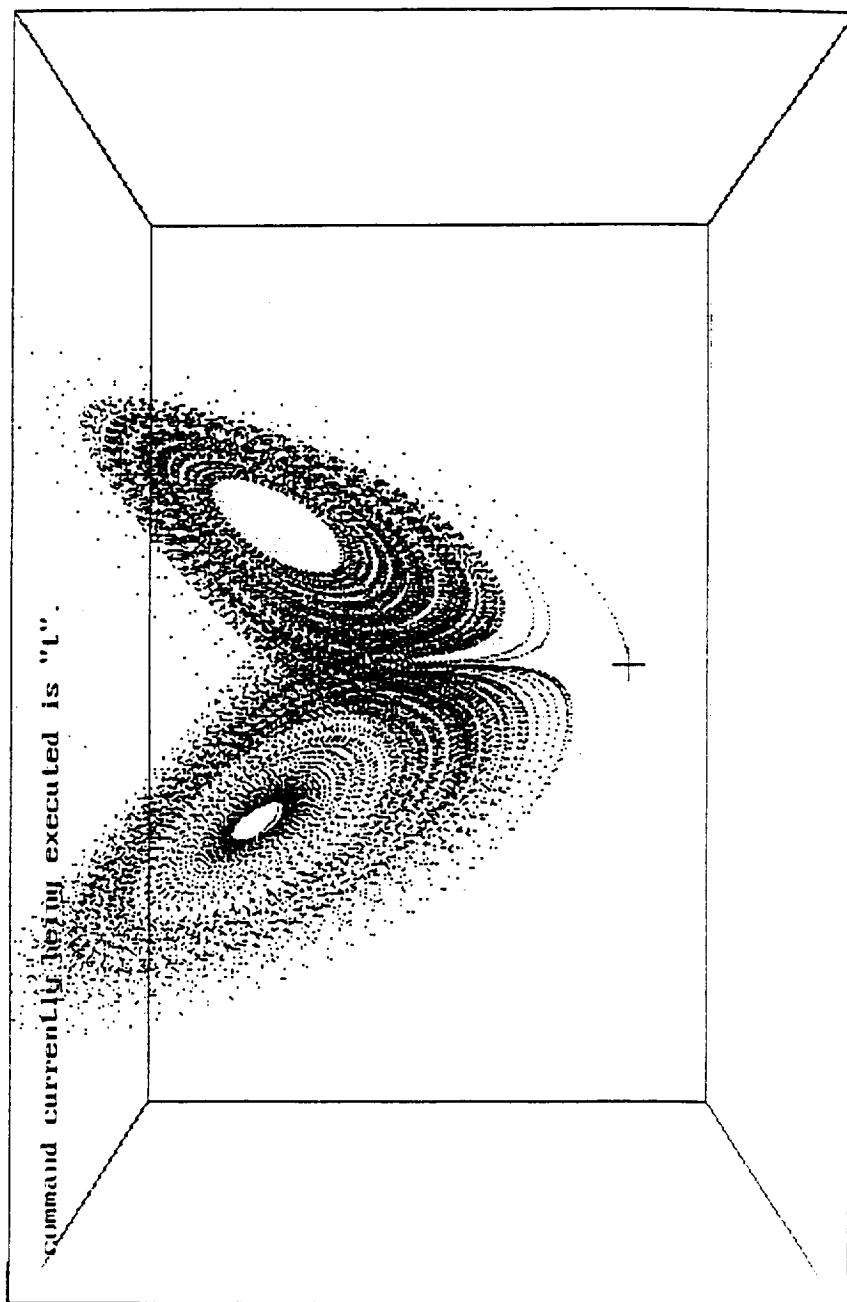


Figure 5-5. The Attractor of the Turbulent Dynamo Constructed Using Our Canonical Transformation



## GLOSSARY

FB	fractional Brownian noise
KS	Schatten solar prediction model
SA	chaotic prediction model



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